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A Parabolic Method without Pressure Approximations for a Wind Farm

Anshul Mittal, W. Roger Briley, Lafayette Taylor and Kidambi Sreenivas

University of Tennessee at Chattanooga, Graduate School for Computational Engineering, SimCenter

SimCenter National Center for Computational Engineering

Abstract

- Issues with existing approaches in predicting wakes
  i. Navier-Stokes simulations (blade resolved or Actuator Disk/Line), though accurate, are computationally impractical for industrial applications.
  ii. Analytical wake models have small runtimes but have reduced accuracy and do not account for all the relevant physics.
- Potential solution

A numerical simulation using a parabolized Navier-Stokes formulation has the capability to offer high-resolution results for forecasting/micro-siting purposes in a reasonable amount of time on a commodity desktop computer.

A new parabolized Navier-Stokes (PNS) formulation for wind turbines is developed based on the three-dimensional viscous primary/secondary flow approximation[1]. This model predicts the wake and secondary-flow velocity fields consistent with the axial momentum and streamwise vorticity equations. The streamwise pressure-gradient field, consistent with these primary and secondary flows, is a dependent variable of the nonlinear modeling equations without further approximations. Turbines are modeled as Actuator Disks with forces computed using NREL's FAST[2] code and imposed as source terms. The solution is initialized well upstream of a turbine and spatially marched through the near-wake and far-wake regions.

Methods

The key to the parabolized Navier-Stokes model is in the viscous and inviscid approximations used. Viscous terms representing streamwise diffusion are neglected. The inviscid approximation is based on a vector decomposition of the transverse velocity field into rotational and irrotational component vectors. The equations are parabolized by neglecting the potential components in the transverse momentum equations that govern the streamwise vorticity, based on order-of-magnitude estimates analogous to those of boundary layer theory but extended to three-dimensional flows with large secondary velocity. The final equation set of five equations is solved by spatial marching using a sequentially block-decoupled, semi-implicit algorithm.

$$\vec{U} = \vec{U}_p + \vec{V}_s$$

$$\vec{U}_p = u\hat{i} \quad , \quad \vec{V}_s = v\hat{j} + w\hat{k}$$

$$\vec{V}_s = \nabla_s \phi + \nabla_s \times \left(\hat{i}\psi\right) = \vec{V}_\phi + \vec{V}_\psi$$

$$\vec{V}_\phi = \hat{j} v_\phi + \hat{k} w_\phi = \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

$$\vec{V}_\psi = \hat{j} v_\psi + \hat{k} w_\psi = \hat{j}\frac{\partial \psi}{\partial z} - \hat{k}\frac{\partial \psi}{\partial y}$$

$$\vec{U}\cdot\nabla\Omega_x + NL = \frac{1}{Re}\left(\frac{\partial^2 \Omega_x}{\partial y^2} + \frac{\partial^2 \Omega_x}{\partial z^2}\right) + \frac{\partial b_z}{\partial y} - \frac{\partial b_y}{\partial z}$$

$$\left(\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) = -\Omega_x$$

$$\left(\frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right) = -\frac{\partial}{\partial y}\left(\vec{U}\cdot\nabla\right)v_\psi - \frac{\partial}{\partial z}\left(\vec{U}\cdot\nabla\right)w_\psi + \frac{\partial b_y}{\partial y} + \frac{\partial b_z}{\partial z}$$

$$\vec{U}\cdot\nabla u + \frac{\partial\left(p/\rho\right)}{\partial x} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + b_x$$

$$\left(\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) = -\frac{\partial u}{\partial x}$$

$$\text{where } NL = \left(\frac{\partial u}{\partial y}\frac{\partial w_\psi}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial w_\psi}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial w_\psi}{\partial z}\right) - \left(\frac{\partial u}{\partial z}\frac{\partial v_\psi}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial v_\psi}{\partial y} + \frac{\partial w}{\partial z}\frac{\partial v_\psi}{\partial z}\right)$$

The Parabolized Navier-Stokes (PNS) approximation for the incompressible equations is combined with the state-of-the-art wind turbine model to simulate flows through wind turbines. The PNS formulation differs from commonly used parabolic formulations in that the entire pressure field is calculated as a dependent variable without any approximation for the streamwise pressure gradient. The turbine influence is modeled by localized source terms that incorporate time-averaged aerodynamic forces predicted by an Actuator Line model (FAST[2] developed at NREL). This coupled time-averaged spatial-marching method allows the calculation to begin well upstream of the turbine and continue through the turbine to predict the entire near- and far-wake region. For a wind farm, the prevailing wind defines the primary-flow direction and the solution is spatially marched in this direction.

$$\vec{f} = \vec{F}\exp\left[-x_D^2\right]\exp\left[-\left(r/\epsilon\right)^2\right]$$

r is the distance between a computational node and a polar grid node (where forces are stored)

ε is the projection width to nondimensionalize the distance r

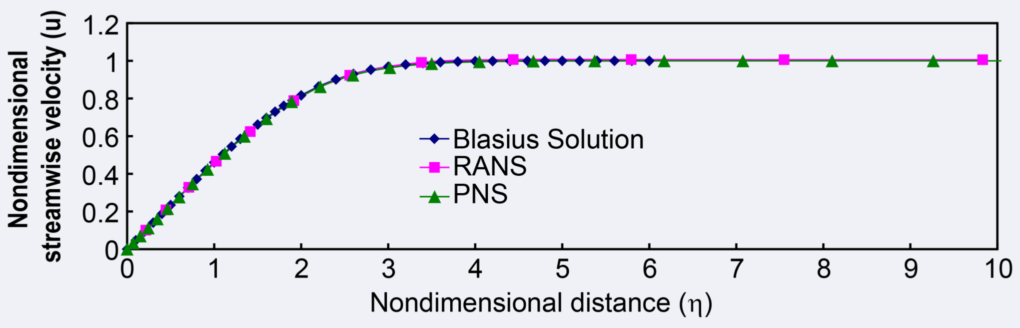
F is the time-averaged force computed from FAST

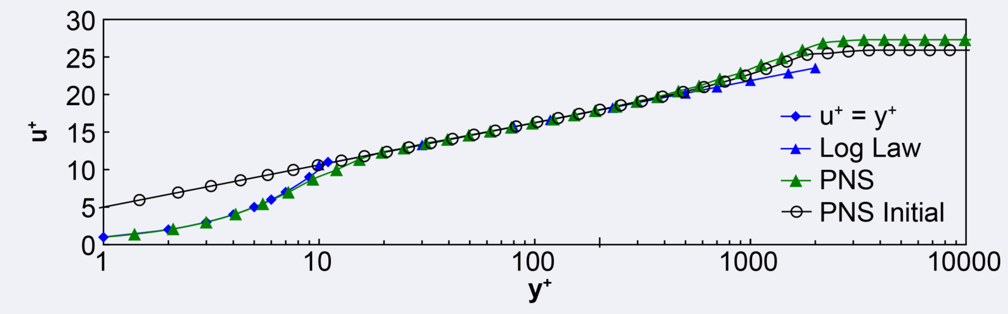
f is the force applied at each computational node in the PNS code

x_D is the distance from the plane in rotor diameters to the wind turbine

Results

Laminar flow over a flat plate: Laminar flow over a flat is simulated for a Reynolds number of 20,000 with a marching step size of 0.01 m. The computations were conducted for a total of 200 steps and the final solution at x=2 m is shown along with the RANS solution and the Blasius solution.



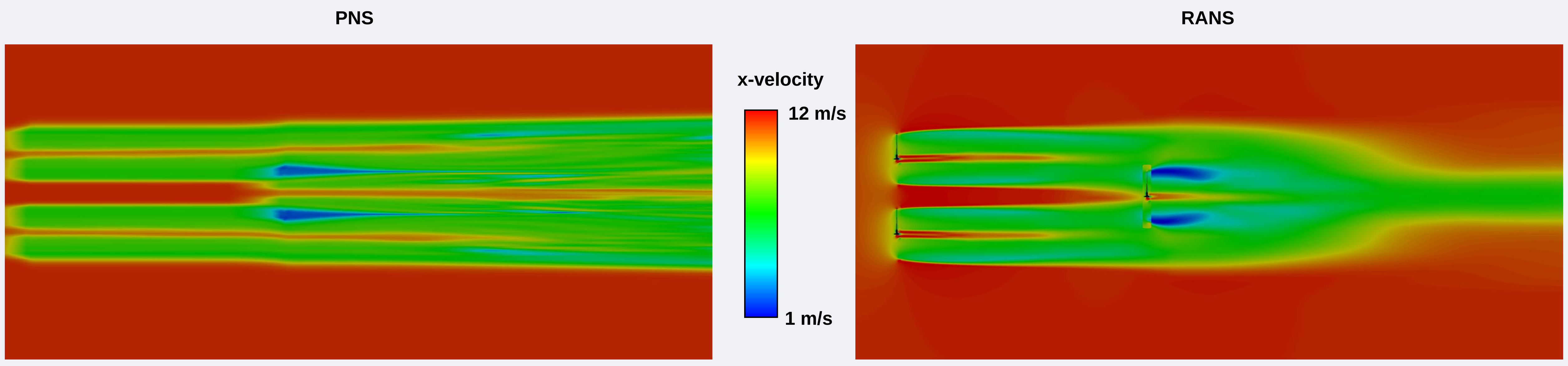


Turbulent flow over a flat plate: Spalart-Allmaras model is utilized for turbulence closure. The Reynolds number is 10^6 and the marching step size is 0.01 m. The initialized solution at x=5.01 m and the computed solution at x=6 m are plotted against the law of the wall.

Convection of a vortex: A streamwise vortex is created by imposing forces in the transverse plane (y-z plane). The vortex is convected downstream (+x) by a uniform axial flow. The same flow conditions are simulated using a Navier-Stokes code with the same forces applied as source terms to generate the vortex. The PNS solution is compared against the Navier-Stokes solutions below.

Location (m)	RANS 2nd order accurate		RANS 5th order accurate		RANS 7th order accurate		RANS 7th order accurate (finer grid)		PNS	
	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.
y-velocity										
0.2	5.184	-5.313	5.372	-5.603	5.271	-5.698	5.162	-5.359	5.584	-5.695
1.0	3.994	-3.995	4.901	-5.031	5.135	-5.466	5.248	-5.262	5.585	-5.692
1.8	3.336	-3.318	4.484	-4.553	4.884	-5.116	5.255	-5.357	5.595	-5.692
Streamwise vorticity										
0.2	0.8014	-2.948	0.8011	-3.104	0.8214	-3.124	1.810	-6.030	1.109	-3.999
1.0	0.7460	-2.162	0.7090	-2.818	0.6874	-3.051	1.594	-5.315	1.110	-3.993
1.8	0.6604	-1.780	0.7324	-2.561	0.6970	-2.949	1.449	-4.699	1.109	-3.987

Array of wind turbines: An array of three NREL 5 MW wind turbines is simulated. Two of these turbines are placed upstream with a lateral spacing of 1.5 rotor diameters. The third turbine is located 5 rotor diameters downstream of these turbines and in the middle so that it is affected by the wakes of both the upstream turbines. All the three turbines operated at 12.1 rpm and the simulation was conducted for a uniform inflow wind speed of 11.4 m/s. The x-velocity contour plot of the PNS solution showing the wakes is given below. For verification purposes, the same configuration was simulated using a blade-resolved Navier-Stokes code and the x-velocity contour plot is also shown below.



The computational cost of the PNS solution on a desktop computer was little over 2 hours whereas the cost of the Navier-Stokes simulation was over 120 hours and required 500 cores of a supercomputer/cluster.

Conclusions

The developed PNS model has been validated for several test cases including a single wind turbine[3]. The low computational cost with the high-fidelity simulation capabilities make the model suitable for industrial applications. The PNS model is under-development and capabilities to incorporate turbulence and atmospheric boundary layer will be added.

References

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2. Jonkman, J. M., and Buhl Jr., M. L., "FAST User's Guide," NREL/EL-500-29798, Golden, CO, 2005.
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Acknowledgments

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