A Unified Method for Probabilistic Forecast of Wind Power Generation

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Outline

1. Introduction
2. Data
3. Motivation
4. SDE-models
5. Time varying parameters
6. Wind Speed Models
7. Summary and conclusion
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Motivation

An increasing part of electricity supply is generated by wind
- Wind power cover about 29% of total system load
- Renewables should cover 50% in 2020 and 100% of total system load in 2035

With the large penetration of wind accurate forecasts (including uncertainties) are needed on all timescales
- minutes - few hour: efficient and safe regulation
- 12-36 hour: efficient trading on NordPool
- days: optimal regulation of large CHP

We focus on horizons from 1-48 hours.
Methods in use

- Adaptive time series model, using MET-forecast (e.g. WPPT)
- Regime models (SETAR, STAR, MSAR)
- Spatio-temporal models
- Combining several MET-forecast
- Corrected MET ensembles (uncertainty)
- Time-adaptive quantile regression (uncertainty)
- Scenario based forecasting (dependence structure by correlation matrix or copula)
- Stochastic differential equations

Most methods are implemented in Wind Power Prediction Tool (WPPT).
WPPT point-forecast

WPPT provide a point forecast

\[
\hat{p}_{t+k|t} = \sum_{i=0}^{n_a} a_i p_{t-i} + b \hat{p}_{t+k|t}^{pc} (MET) + f(h_{t+k})
\]

where \( p_t \) is observed power production, \( k \in [1; 48) \) prediction horizon, \( \hat{p}_{t+k|t}^{pc} (MET) \) is a power curve prediction and \( h_{t+k} \) is time of day.

- Parameters are estimated adaptively
- WPPT is one of the most widely used forecasting tools for windpower (worldwide)
- WPPT point forecasts are used as input to SDE-models
Obs. \( \hat{p}_{t|0} \)

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5, 2003, 18h</td>
<td></td>
</tr>
<tr>
<td>Oct. 22, 2002, 00h</td>
<td></td>
</tr>
<tr>
<td>April 22, 2001, 12h</td>
<td></td>
</tr>
<tr>
<td>May 19, 2001, 12h</td>
<td></td>
</tr>
</tbody>
</table>

Power

Time
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The data set cover the period from January 1, 2001 to May 2, 2003.

- Hourly measurements of actual power production
- 48 hour point-forecast of power production (issued at 00h, 06h, 12h, 18h)
- Only sets where all point forecasts and all measurements are non-missing are used in this analysis (2593 complete sets in total)
- 150 sets are used to train the SDE models
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Correlation

\[
p^{t+1|0} - p^t + 1
\]

Obs. BM 0

BM 1
BM 3 - standard deviation and correlation

![Graph showing standard deviation and correlation](image_url)
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Scope

The scope of the SDE-modelling is to generate covariance structures based on SDE formulations. Given the (continuous time) SDE-formulation

\[ dx_t = f(x_t, u_t, \theta) dt + \sigma(x_t, u_t, \theta) dw_t; \quad x_0 \text{ given} \]

and the (discrete time) observation equation

\[ p = h(x, u, \theta) + e; \quad e \sim N(0, S) \]

where

- \( p \in [0, 1]^{48} \) is observed power
- \( x = \{x_1, ..., x_{48}\} \) is the state vector
- \( u_t \) is some input (here predicted power)
A SDE-formulation for error propagation

The starting point (the Pearson/logistic diffusion)

$$dx_t = -\theta \cdot (x_t - \mu)dt + \sqrt{2\theta ax_t \cdot (1 - x_t)}dw_t,$$

with $\mu \in (0, 1)$, $a < \min(\mu, 1 - \mu)$.

- $x_t \in (0, 1)$
- Diffusion (variance) small when $x_t$ is close to 0 or 1.
- Long term average equal $\mu$
- Stationary distribution is a beta distribution (with parameters $\frac{\mu}{a}$ and $\frac{1-\mu}{a}$)
- Interdependence structure controlled by $\theta$
Basic models

The second order moment representation can be solved for \( a \in (0, 2) \), and we choose

\[
dx_t = -\theta \cdot (x_t - \mu - \gamma (1 - 2x_t))dt + 2\sqrt{\theta ax_t \cdot (1 - x_t)}dw_t,
\]

with the observation equation given by

\[
y = x + e; \quad e \sim N(0, \Sigma),
\]

where

- \( \mu = \hat{p}_{t|0} \) (mean)
- \( a = \alpha \hat{p}_{t|0,i} (1 - \hat{p}_{t|0,i}) \) with \( \alpha \in (0, 1) \) (variance)
- \( \gamma = c \hat{p}_{t|0,i} (1 - \hat{p}_{t|0,i}) \) with \( c \in \mathbb{R}_+ \) (bias)
## Results

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>l(train)</th>
<th>p-value</th>
<th>l(test1)</th>
<th>l(test2)</th>
<th>l(test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM 2</td>
<td>7</td>
<td>8393</td>
<td></td>
<td>88690</td>
<td>67777</td>
<td>156467</td>
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<td>BM 3</td>
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<td>8419</td>
<td>&lt; 0.0001</td>
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<td>67853</td>
<td>156546</td>
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<td>SDE 0</td>
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<td>161598</td>
</tr>
<tr>
<td>SDE 1</td>
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<td>8532.4</td>
<td>&lt; 0.0001</td>
<td>94292</td>
<td>70719</td>
<td>165011</td>
</tr>
</tbody>
</table>

*SDE0*: No bias  
*SDE1*: Including bias
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Models

Model of increasing complexity are analysed, s is (an estimated constant) in all models. Non-linear relations are explored by

\[
\alpha(\hat{p}_t|0, t) = \frac{1}{1 + \exp(-\alpha_0 - f_\alpha(\hat{p}_t|0) - g_\alpha(t))}
\]

\[
\theta(\hat{p}_t|0, t) = \frac{K_\theta}{1 + \exp(-\theta_0 - f_\theta(\hat{p}_t|0) - g_\theta(t))}
\]

\[
c(\hat{p}_t|0, t) = \frac{K_c}{1 + \exp(-c_0 - f_c(\hat{p}_t|0) - g_c(t))}
\]

the non-linear functions in \( f \) and \( g \) are modelled by natural cubic splines. \( K_\theta \) and \( K_c \) are used to control the range of \( c \) and \( \theta \), we use \( (K_\theta = K_c = 20) \).
Standard deviation

Time [h]

SDE 5

SDE 6

$\hat{p}_{t|0}$
Ensemble forecasts
Time varying parameters

Obs.  \( \hat{p}_{t|0} \)  \( \hat{p}_{SDE} \)

May 5. 2003, 18h  Oct. 22. 2002, 00h

April 22. 2001, 12h  May 19. 2001, 12h

Power

Time

0.0  0.2  0.4  0.6  0.8  1.0

12  24  36  48
Time varying parameters

- April 22, 2001, 12h
- May 5, 2003, 18h
- Oct. 22, 2002, 00h
- May 19, 2001, 12h
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A SDE model for wind speed variability.

We start out with a simple SDE model. Instead of predicted power output, the numerical weather prediction (NWP) is used:

$$dx_t = -\theta \cdot (x_t - \text{NWP}_t)dt + \sigma \cdot x_t^{\gamma} \cdot dw_t.$$  

As there is not physical upper bound for the wind speed as opposed to wind power, we choose the diffusion term $g(x, t) = \sigma \cdot x^{\gamma}$. 
Predictive Density, 1h & 5h
Introducing the derivative of the prediction

We now introduce the derivative of the NWP with respect to time, NWP, to the model:

\[ dx_t = (-\theta_1 \cdot (x_t - \text{NWP}_t) + \theta_2 \cdot (1 - e^{-x_t}) \cdot \text{NWP}) dt + \sigma \cdot x_t^\gamma \cdot dw_t. \]

Here we have introduced the term \((1 - e^{-x_t})\) for the model to remain feasible, as it makes sure that the process remains in \(\mathbb{R}^+\), as the influence of NWP drops to zero when \(x_t\) approaches zero.
Predictive Density, 1h & 5h
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We model wind power production as multivariate Gaussian (possibly after some transformation), and focus on the covariance structure.

- SDEs are used to generate very flexible covariance structures.
- The SDEs model quite complex covariance structures.
- Ensemble forecast and prediction densities (and intervals) are easily obtained from SDE’s.
- Introducing the derivative of NWP w.r.t. time, gave large improvements in terms of predictive densities and time lags.