A Unified Method for Probabilistic Forecast of Wind Power Generation

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Outline

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Motivation

An increasing part of electricity supply is generated by wind

- Wind power cover about 29% of total system load
- Renewables should cover 50% in 2020 and 100% of total system load in 2035

With the large penetration of wind accurate forecasts (including uncertainties) are needed on all timescales

- minutes - few hour: efficient and safe regulation
- 12-36 hour: efficient trading on NordPool
- days: optimal regulation of large CHP

We focus on horizons from 1-48 hours.

Methods in use

- Adaptive time series model, using MET-forecast (e.g. WPPT)
- Regime models (SETAR, STAR, MSAR)
- Spatio-temporal models
- Combining several MET-forecast
- Corrected MET ensembles (uncertainty)
- Time-adaptive quantile regression (uncertainty)
- Scenario based forecasting (dependence structure by correlation matrix or copula)
- Stochastic differential equations

Most methods are implemented in Wind Power Prediction Tool (WPPT).

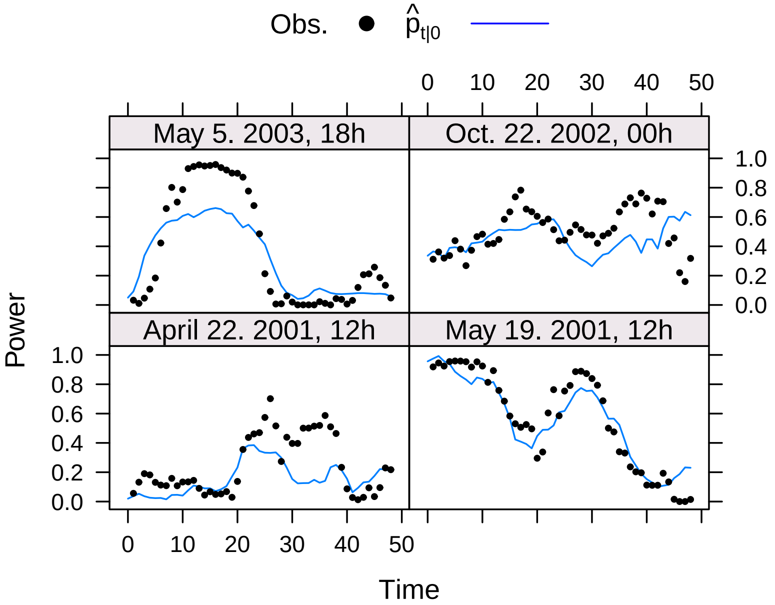
WPPT point-forecast

WPPT provide a point forecast

$$\hat{p}_{t+k|t} = \sum_{i=0}^{n_a} a_i p_{t-i} + b\hat{p}^{pc}_{t+k|t}(MET) + f(h_{t+k})$$

where p_t is observed power production, $k \in [1;48)$ prediction horizon, $\hat{p}^{pc}_{t+k|t}(MET)$ is a power curve prediction and h_{t+k} is time of day.

- Parameters are estimated adaptively
- WPPT is one af the most widely used forecasting tools for windpower (worldwide)
- WPPT point forecasts are used as input to SDE-models



Obs.
$\hat{p}_{t|0}$
May 5. 2003, 18h
Oct. 22. 2002, 00h
April 22. 2001, 12h
May 19. 2001, 12h
Power
Time
0
10
20
30
40
50
0.0
0.2
0.4
0.6
0.8
1.0

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Data

The data set cover the period from January 1. 2001 to May 2. 2003.

- Hourly measurements of actual power production
- 48 hour point-forecast of power production (issued at 00h, 06h, 12h, 18h)
- Only sets where *all point forecasts and all measurements* are non-missing are used in this analysis (2593 complete sets in total)
- 150 sets are used to train the SDE models

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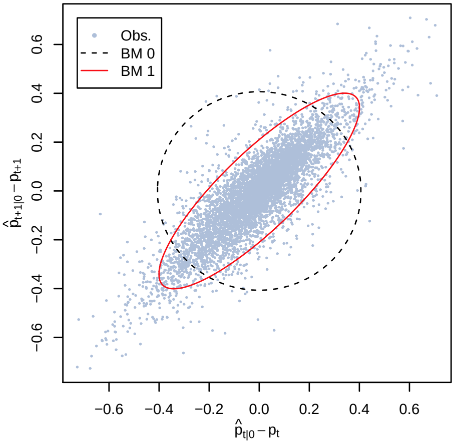
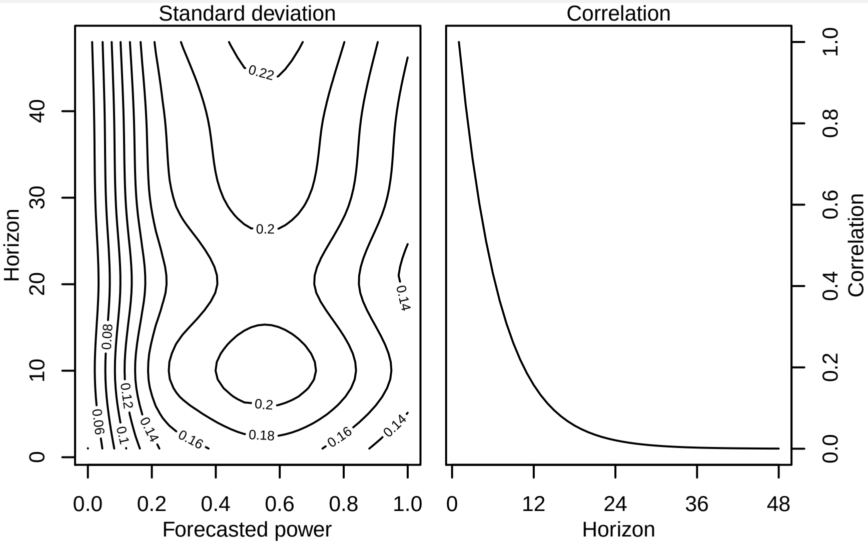
Correlation



BM 3 - standard deviation and correlation



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Scope

The scope of the SDE-modelling is to generate covariance structures based on SDE formulations.

Given the (continuous time) SDE-formulation

$$dx_t = f(x_t, u_t, \boldsymbol{\theta})dt + \sigma(x_t, u_t, \boldsymbol{\theta})dw_t; \quad x_0 \text{ given}$$

and the (discrete time) observation equation

$$\boldsymbol{p} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta}) + \boldsymbol{e}; \qquad \boldsymbol{e} \sim N(\boldsymbol{0}, \boldsymbol{S})$$

where

- $\boldsymbol{p} \in [0, 1]^{48}$ is observed power
- $\boldsymbol{x} = \{x_1, ..., x_{48} | x_0\} \in [0, 1]^{48}$, is the state vector
- u_t is some input (here predicted power)

A SDE-formulation for error propagation

The starting point (the Pearson/logistic diffusion)

$$dx_t = -\,\theta \cdot (x_t - \mu)dt + \sqrt{2\theta a x_t \cdot (1 - x_t)}dw_t,$$

with $\mu \in (0, 1)$, $a < min(\mu, 1 - \mu)$.

- $x_t \in (0, 1)$
- Diffusion (variance) small when x_t is close to 0 or 1.
- Long term average equal μ
- Stationary distribution is a beta distribution (with parameters $\frac{\mu}{a}$ and $\frac{1-\mu}{a}$)
- Interdependence structure controlled by θ

Basic models

The second order moment representation can be solved for $a \in (0, 2)$, and we choose

$$dx_t = -\theta \cdot (x_t - \mu - \gamma(1 - 2x_t))dt + 2\sqrt{\theta a x_t \cdot (1 - x_t)}dw_t,$$

with the observation equation given by

$$\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{e}; \quad \boldsymbol{e} \sim N(\boldsymbol{0}, \boldsymbol{S}),$$

where

- $\mu = \hat{p}_{t|0}$ (mean)
- $a = \alpha \hat{p}_{t|0,i}(1 - \hat{p}_{t|0,i})$ with $\alpha \in (0, 1)$ (variance)
- $\gamma = c\hat{p}_{t|0,i}(1 - \hat{p}_{t|0,i})$ with $c \in \mathbb{R}_+$ (bias)

Results

	df	l(train)	p-value	l(test1)	l(test2)	l(test)
BM 2	7	8393		88690	67777	156467
BM 3	12	**8419**	< 0.0001	**88692**	**67853**	**156546**
SDE 0	3	8408.4		92376	69222	161598
SDE 1	4	**8532.4**	< 0.0001	**94292**	**70719**	**165011**

$$SDE0 : \text{No bias}$$

$$SDE1 : \text{Including bias}$$

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Models

Model of increasing complexity are analysed, s is (an estimated constant) in all models. Non-linear relations are explored by

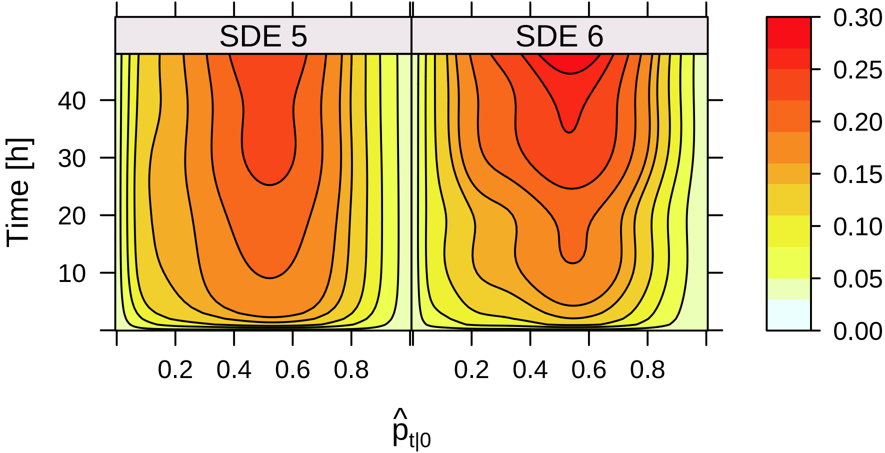
$$\alpha(\hat{p}_{t|0}, t) = \frac{1}{1 + exp(-\alpha_0 - f_\alpha(\hat{p}_{t|0}) - g_\alpha(t))}$$

$$\theta(\hat{p}_{t|0}, t) = \frac{K_\theta}{1 + exp(-\theta_0 - f_\theta(\hat{p}_{t|0}) - g_\theta(t))}$$

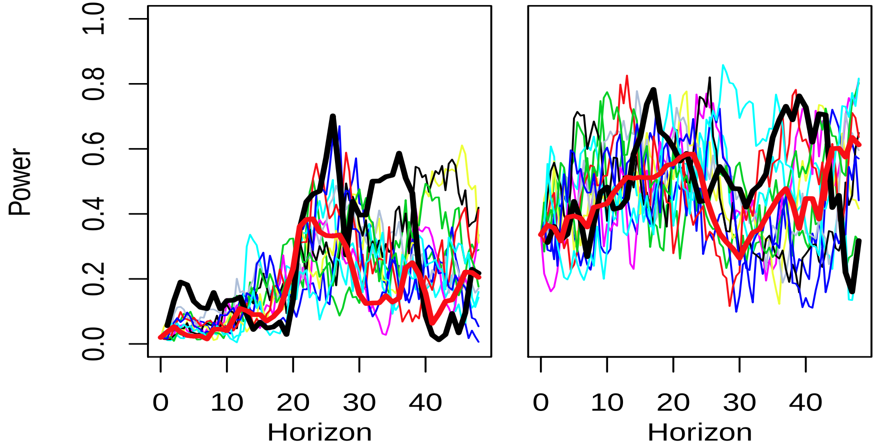
$$c(\hat{p}_{t|0}, t) = \frac{K_c}{1 + exp(-c_0 - f_c(\hat{p}_{t|0}) - g_c(t))}$$

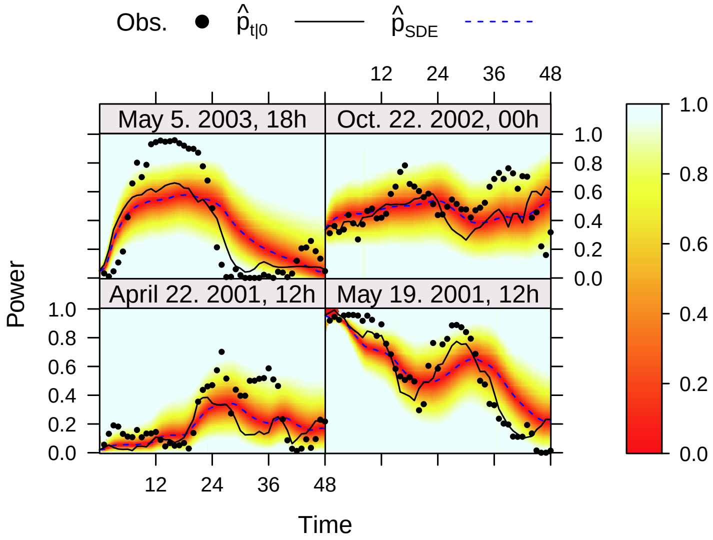
the non-linear functions in f and g are modelled by natural cubic splines. K_θ and K_c are used to control the range of c and θ, we use $(K_\theta = K_c = 20)$.

Standard deviation

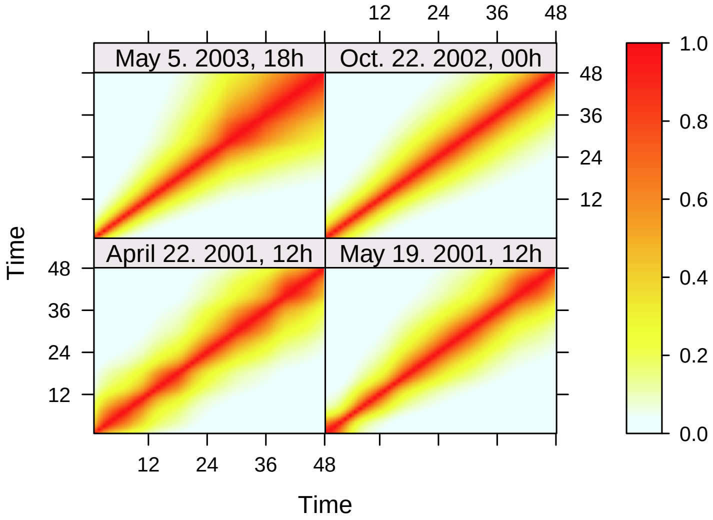


Ensemble forecasts





Obs.
$\hat{p}_{t|0}$
$\hat{p}_{SDE}$
May 5. 2003, 18h
Oct. 22. 2002, 00h
April 22. 2001, 12h
May 19. 2001, 12h
Power
Time
12
24
36
48
0.0
0.2
0.4
0.6
0.8
1.0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0.4
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0.0

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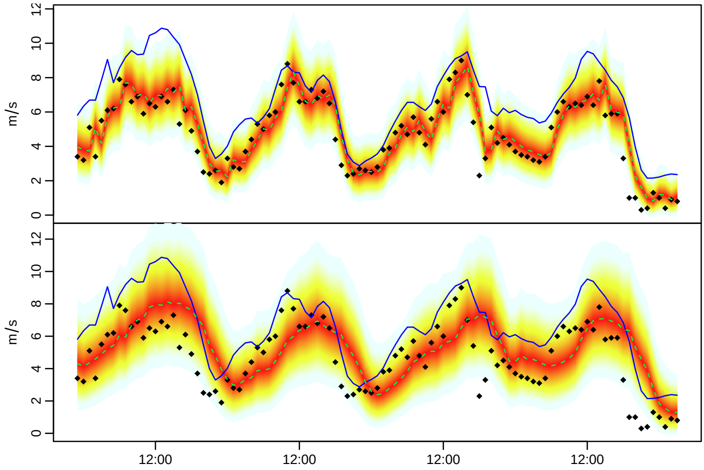
A SDE model for wind speed variability.

We start out with a simple SDE model. Instead of predicted power output, the numerical weather prediction (NWP) is used:

$$dx_t = -\,\theta \cdot (x_t - \mathrm{NWP}_t)dt + \sigma \cdot x_t^{\gamma} \cdot dw_t.$$

As there is not physical upper bound for the wind speed as opposed to wind power, we choose the diffusion term $g(x,t) = \sigma \cdot x^{\gamma}$.

Predictive Density, 1h & 5h



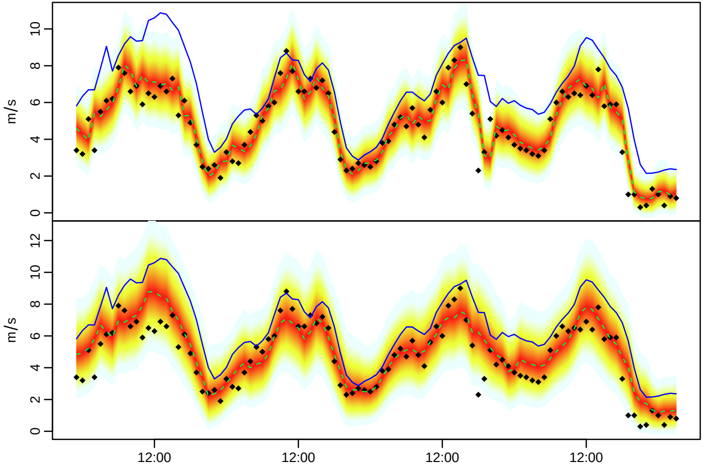
Introducing the derivative of the prediction

We now introduce the derivative of the NWP with respect to time, $\dot{\mathrm{NWP}}$, to the model:

$$dx_t = (-\theta_1 \cdot (x_t - \mathrm{NWP}_t) + \theta_2 \cdot (1 - e^{-x_t}) \cdot \dot{\mathrm{NWP}})dt + \sigma \cdot x_t^{\gamma} \cdot dw_t.$$

Here we have introduced the term $(1 - e^{-x_t})$ for the model to remain feasible, as it makes sure that the process remains in $\mathbb{R}^+$, as the influence of $\dot{\mathrm{NWP}}$ drops to zero when x_t approaches zero.

Predictive Density, 1h & 5h



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Summary and conclusion

- We model wind power production as multivariate Gaussian (possibly after some transformation), and focus on the covariance structure
- SDEs are used to generate very flexible covariance structures
- The SDEs model quite complex covariance structures
- Ensemble forecast and prediction densities (and intervals) are easily obtained from SDE's
- Introducing the derivative of NWP w.r.t. time, gave large improvements in terms of predictive densities and time lags.