Symbolic Solution Approach to Wind Turbine based on Doubly Fed Induction Generator Model

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***Index Terms*—Symbolic computation, wind turbine modelling, voltage dip, DFIG**

NOMENCLATURE

H_{eq}	Moment of inertia of entire wind turbine.
H_g	Moment of inertia of generator.
$\boldsymbol{i_s}, \boldsymbol{i_r}$	Stator and rotor current vectors.
K_{opt}	Optimal torque/speed constant.
N_{gb}	Gear box ratio.
L_s	Stator inductance.
L_r	Rotor inductance.
L_m	Magnetizing inductance.
p	Number of pair of poles.
r	Turbine blade radius.
R_s	Stator resistance.
R_r	Rotor resistance.
S_b	Power base value.
$\boldsymbol{u_s}, \boldsymbol{u_r}$	Stator and rotor voltage vectors.
v	Wind speed.
T_e	Electromagnetic torque.
T_m	Mechanical and electromagnetic torque.
β	Blade pitch angle.
λ	Stip speed ratio.
ρ	Air density.
$\boldsymbol{\psi_s}$	Stator flux linkage vectors.
$\boldsymbol{\psi_r}$	Rotor flux linkage vectors.
ω_{eb}	Electrical angular speed base value.
ω_s	Synchronous angular speed.
Ω_g	Generator rotational speed.
Ω_h	Hub mechanical rotational speed.
s, r	First subscript indicates stator and rotor.
g, gc	First subscript indicates grid and grid-converter.
d, q	Second subscript indicates direct and quadrature axes quantities.

I. INTRODUCTION

This paper describes an alternative approach based on symbolic computations to simulate wind turbines equipped with Doubly–Fed Induction Generator (DFIG). The actuator disk theory is used to represent the aerodynamic part, and the one-mass model simulates the mechanical part. The 5^{th}–order induction generator is selected to model the electric machine, being this approach suitable to estimate the DFIG performance under transient conditions. The corresponding non–linear integro-differential equation system has been reduced to a linear state-space system by using an ad-hoc local linearization. This novel Symbolic Computation (SYMB) method has been implemented by using two different software-packages, with the purpose of solving simultaneously a remarkable number of individual wind turbines models submitted to different wind speed profiles and/or grid voltage waveforms.

The obtained results are compared with traditional Finite Difference Discretization (FDD) methods, widely proposed for this type of studies. The results offer a good agreement between the proposed SYMB method and the FDD solutions, considering variations of both wind speed profiles and electrical transient events.

II. APPROACH

These days, the number of wind turbines connected to power systems requires a special attention from the Transmission and System Operators (TSOs). Advanced models of wind farms are demanded by current TSOs for planning and operating purposes. However, accurate simulations of wind turbines usually imply significant computational efforts. For that reason, most previous contributions have been mainly focused on aggregation techniques by reducing the wind farms to an equivalent wind turbine model, [1],[2], [3], [4] and [6]. In [5], a wind farm of 12 full-converter wind turbines

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using permanent magnet synchronous machines with a rated power of 5 MW is simulated. In this case, the wind farm is divided into three groups, and each group is formed by the wind turbines receiving a similar wind speed profiles. Consequently, each group is then reduced to an equivalent wind turbine. In [7], the mechanical characteristics of the wind turbine, the electro-mechanical parameters of the generator and the converters are aggregated to represent the equivalent wind farm model. In [8], a wind farm consisting of 68 DFIG wind turbines is modeled and simulated in the same way.

With regard to wind turbine simulations, most commercial software packages proposed for this type of studies are focused on the electrical part, being simplified both aerodynamic and mechanical parts. These software packages usually involve numerical methods, mainly Finite Difference Discretization (FDD) techniques, (PSS/E, Power Factory–DIgSILENT or PSCAD/EMTDC), [9]. However, some drawbacks have been detected when a significant number of individual wind turbines are simultaneously simulated, mainly (i) excessive computational time, (ii) memory requirements or number of variables. For that reason and as was previously discussed, wind turbines are usually aggregated as an equivalent wind turbine, [10], [11]. Analytic techniques are an alternative to the FDD approach, [9]. However, during the last decades their use has been partially discarded, mainly due to the non-linearity of the wind turbine models as well as the computational capacity limitations of the symbolic math software packages. In fact, few contributions can be found in the specific literature regarding these techniques,[12], [13] and [15].

III. MAIN BODY OF ABSTRACT

A. Wind Turbine Modeling

The basic scheme of a wind turbine equipped with DFIG is represented in Fig. 1. In this configuration, the stator terminals are directly connected to the grid and the rotor terminals are connected through a back-to-back converter, which size is determined for its capacity of handling around 25–30% of the rated power of the wind turbine [19]. The pitch angle β is fixed to zero in all simulations. The wind turbine model proposed in this paper involves the following conditions and assumptions [20]:

- All quantities are referred to the stator-side and taken in per unit, (pu), except ω_{eb} that is in electrical rad/s and t is in seconds.
- The stator current is considered as a positive value when flowing towards the machine, since traditionally the proposed induction machine models have been studied in motor mode [21].
- The q-axis is assumed to be $\pi/2$ ahead of the d-axis with respect to the direction of rotation. Both d and q windings are magnetically decoupled, allowing to control independently active and reactive power variables [22], [23].
- The (d, q) reference system rotates at the same speed value and direction as the stator flux $\boldsymbol{\psi_s}$ (corresponding to the grid frequency speed), becoming the stator parameters (voltage, current and flux) close to their steady-state values.

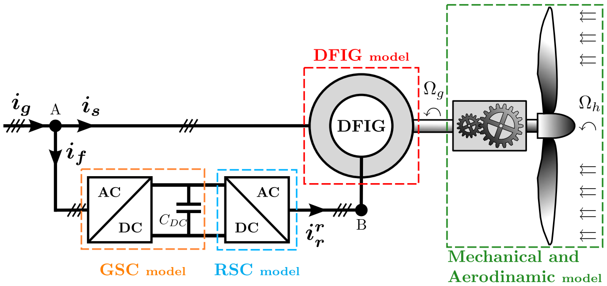

Fig. 1. Basic scheme of a DFIG wind turbine

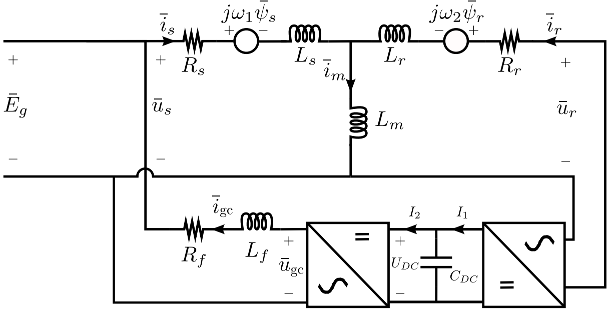

Fig. 2. Wind turbine electrical equivalent circuit considering DFIG 5^{th} order (all quantities are referred to the stator-side).

The wind turbine model considered in this work is based on the model developed in [24] but with some differences: in the present work the Grid Side Converter control, [33], and 5^{th} order of the DFIG has been implemented, while in [24] was not considered the GSC control and for the DFIG was utilized the 3^{rd} order model.

B. An approach to a linear wind turbine model

The wind turbine model can be divided into two parts: the electrical part and the aerodynamic-mechanical part. In the next two subsections is discussed how to linearize both parts.

1) Linear state–space model for the electrical part: If $\omega_g(t)$ is assumed as constant along a simulation time interval $\tau_j = [t_{0_j}, t_{f_j}]$, the non–linear integro–differential electrical part model defined in section III-A can be arranged in a linear state–space form. The suitability of this assumption ($\omega_g(t)$ constant) is based on the fact that for power system simulations involving grid disturbances taking time intervals usually lower than 30 seconds, being possible to assume wind speed values as constant [25]. It must be pointed out that this assumption ($\omega_g(t)$ as constant) is only applied for the linearization process of the electrical part, and it is not considered as a constant variable along the whole time interval of the simulation. In fact, the evolution of $\omega_g(t)$ along a $\tau_j = [t_{0_j}, t_{f_j}]$ is obtained by solving the linearized motion equation described in Section III-B2.

The equation-system can be transformed into a differential equation system by extending the number of space-state variables. The following change of variables is proposed, [26], [27], in order to adapt the expresions of the Proportional Integral controllers of the Rotor Side Converter and the Grid Side Converter to the state-space form of the model:

$$\epsilon_{rd} = \int (i_{rd}^{ref} - i_{rd}) \tag{1}$$

$$\epsilon_{rq} = \int (i_{rq}^{ref} - i_{rq}) \tag{2}$$

$$\epsilon_{gcd} = \int (i_{gcd}^{ref} - i_{gcd}) \tag{3}$$

$$\epsilon_{gcq} = \int (i_{gcq}^{ref} - i_{gcq}). \tag{4}$$

A first order linear differential equation system can be then deduced and written as:

$$\boldsymbol{M} \cdot \dot{\boldsymbol{X}}(\boldsymbol{t}) = \boldsymbol{N} \cdot \boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{S} \cdot \boldsymbol{U}(\boldsymbol{t}), \tag{5}$$

$$\dot{\boldsymbol{X}}(\boldsymbol{t}) = \boldsymbol{A} \cdot \boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{F}(\boldsymbol{t}), \tag{6}$$

where $\boldsymbol{A} = \boldsymbol{M^{-1}N}$ and $\boldsymbol{F}(\boldsymbol{t}) = \boldsymbol{M^{-1}S} \cdot \boldsymbol{U}(\boldsymbol{t})$. This rearrangement can be carried out due to the existence of $\boldsymbol{M}$ inverse. Further information about the matrix structure of (6) can be found in the Appendix. The state-space variables $\boldsymbol{X}(\boldsymbol{t})$ and the input vector $\boldsymbol{U}(\boldsymbol{t})$ are respectively,

$$\boldsymbol{X}(\boldsymbol{t}) = [\psi_{sd}, \psi_{sq}, \psi_{rd}, \psi_{rq}, \epsilon_{rd}, \epsilon_{rq}, \epsilon_{gcd}, \epsilon_{gcq}]^T \tag{7}$$

$$\boldsymbol{U}(\boldsymbol{t}) = [u_{sd}, u_{sq}, i_{rd}^{ref}, i_{rq}^{ref}, , i_{gcd}^{ref}, i_{gcq}^{ref}]^T. \tag{8}$$

2) Linear model of aerodynamical-mechanical part: To obtain the analytical expression for the rotational generator speed $\omega_g(t)$ along $\tau_j = [t_{0_j}, t_{f_j}]$ time interval, the motion equation defined in (9) has to be linearized.

$$\frac{d\omega_g}{dt} = \frac{1}{2H} \left(T_m(\omega_g, t) - T_e(t) \right), \tag{9}$$

this non-linear differential equation can be locally linearized as:

$$\frac{d\omega_g}{dt} + G(t) \cdot \omega_g = V(t), \tag{10}$$

with $\omega_{g0_j} = \omega_g(t_{0_j})$ and

$$G(t) = -\left.\frac{\partial f}{\partial \omega_g}\right|_{\omega_{g0_j}, t_{0_j}} ; V(t) = f(\omega_{g0_j}, t_{0_j}) + (t - t_{0_j}) \left.\frac{\partial f}{\partial t}\right|_{\omega_{g0_j}, t_{0_j}} - \omega_{g0_j} \left.\frac{\partial f}{\partial \omega_g}\right|_{\omega_{g0_j}, t_{0_j}} .$$

The inputs to the linearized aerodynamic-mechanical model of the wind turbine are: the profile of wind speed $v(t)$ and the stator and rotor currents $i_{sq}(t), i_{rd}(t), i_{sd}(t), i_{rq}(t)$ expressed in function of time t.

C. Proposed wind turbine model solution

Fig. 3 shows schematically the process proposed for symbolic resolution. The simulated global time interval τ is divided into n time intervals to be solved analytically, $\tau = [\tau_1, \ldots, \tau_n]$. In this case, $\boldsymbol{X}(\boldsymbol{t})$ and $\omega_g(t)$ can be determined for each specific time interval $\tau_j = [t_{0_j}, t_{f_j}]$, $\forall t \in [t_{0_j}, t_{f_j}]$. Regarding ω_{g0_j}, $\boldsymbol{X}(\boldsymbol{t_{0_j}})$ and the input variables of the model ($\boldsymbol{U}$ and the wind speed profile v), they have to be updated each τ_j. The value of ω_{g0_j} for a τ_j time interval (with $j > 1$) is equal to the value of ω_g at the end of the previous interval, $\omega_g(t_{0_j}) = \omega_g(t_{f(j-1)})$, to preserve continuity function properties. A similar process is carried out to obtain the initial conditions of the state-space variables, $\boldsymbol{X}(\boldsymbol{t_{0_j}}) = \boldsymbol{X}(\boldsymbol{t_{f_{j-1}}})$. In our case, these values are known before the initialization of the time interval simulation. Nevertheless, contributions focused on solving initial value problems for a system of linear integro–differential equations can be found in [30].

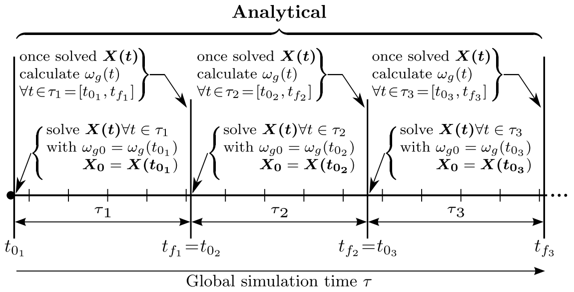

Fig. 3. Scheme of proposed analytical solution considering $\tau = [\tau_1, \tau_2, \tau_3]$

The corresponding state-space system modeling the electrical part, (6), is solved analytically by the method of Variation of Parameters described in [9]. The analytical expression of the solution is:

$$\boldsymbol{X}(\boldsymbol{t}) = \boldsymbol{\phi}(\boldsymbol{t})\boldsymbol{\phi}(\boldsymbol{t_0})^{-1}\boldsymbol{X}(\boldsymbol{t_0}) + \boldsymbol{\phi}(\boldsymbol{t}) \int_{t_0}^{t} \boldsymbol{\phi}^{-1}(\boldsymbol{s})\boldsymbol{F}(\boldsymbol{s}) ds,$$

where $\boldsymbol{\phi}(\boldsymbol{t})$ is named fundamental matrix of the equation system and it can be determined according to [9].

Considering the linearization of the motion equation described in Section III-B2, $\omega_g(t)$ can be determined as the analytical solution of (10) given by [9], $\forall t \in [t_{0_j}, t_{f_j}]$,

$$\omega_g(t) = e^{-\int_{t_{0_j}}^{t} G(t)\, \mathrm{d}t} \cdot \left[\omega_{g0_j} + \int_{t_{0_j}}^{t} V(t)\, \mathrm{d}t \cdot e^{\int_{t_{0_j}}^{t} G(t)\, \mathrm{d}t} \right].$$

Finally, the estimation of active and reactive power for stator, rotor and grid side converter (P_s, P_r, P_{gc}, Q_s, Q_r and Q_{gc}) along the whole time interval can be calculated.

D. Cases study description and results

A set of simulations considering real–measured stator voltage waveforms have been carried out to evaluate the proposed symbolic technique based on symbolic solution and FDD solution. For FDD solution, software package MATLAB–Simulink [31] is used to simulate the wind turbine model according to [29]. Software package Mathematica [14] is selected to solve the process involving symbolic operations. For the rest of the paper, the non–linear wind turbine model solved by FDD approach will be labeled as FDD model, and the linearized wind turbine model solved by symbolic form will be referred as SYMB model.

Case 1 involves a global simulation time of $\tau \approx 30$ seconds. It has been divided into three linearization time intervals with

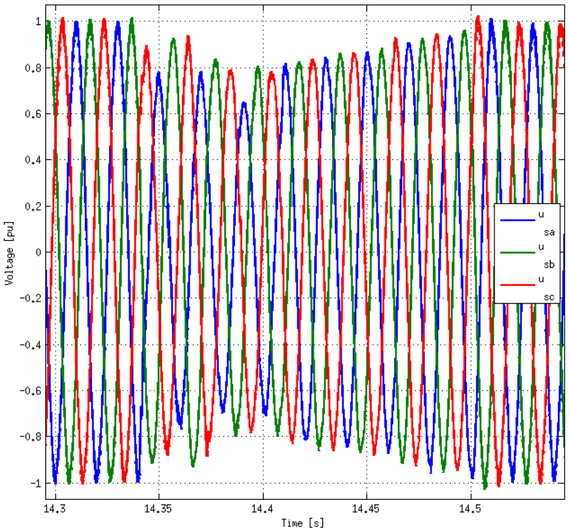

Fig. 4. Stator voltages profile for case of study.

different time durations: $\tau = [14.32, 0.19, 18.99]$. The second time interval involves the voltage dip, [35], and it is illustrated in Fig. III-D in instantaneous values.

The stator currents obtained from FDD and SYMB models are shown in Figs. 5, where the differences along all simulation time between both approaches can be neglected. In Fig. 6, the second time interval along the transient event is depicted in detail. It can be observed how all SYMB values match accurately the FDD values. For the case of rotational generator speed ω_g (see Fig. 7) the differences between FDD and SYMB models are not significant, considering the small values of the differences in per unit. In TableIIIIII are shown the computational costs of FDD and SYMB methods for different number of wind turbines.

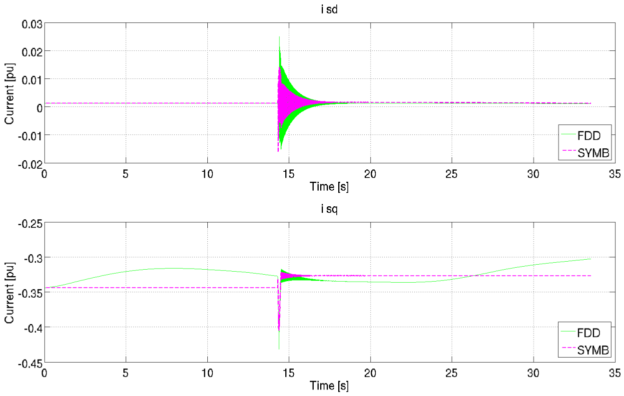

Fig. 5. Stator currents, i_{sd} and i_{sq}, FDD vs SYMB comparison

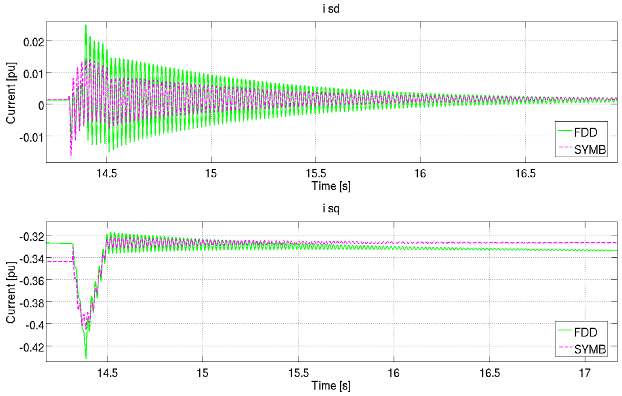

Fig. 6. Stator currents, i_{sd} and i_{sq}, FDD vs SYMB comparison detail

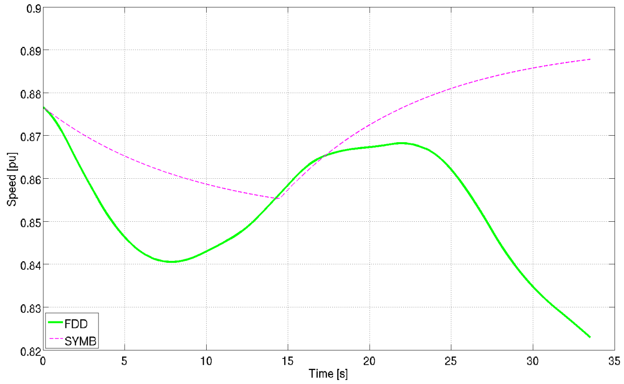

Fig. 7. Generator speed ω_g, FDD vs SYMB comparison

Time interval simulation	**Discretization**	**Computational cost (s)**	
(s)	**Δ_t (s)**	**FDD**	**Symbolic Solution**
14.3	10^{-4}	1.88	0.73
0.2	10^{-4}	0.08	0.56
18.9	10^{-4}	2.45	0.92

TABLE I
COMPUTATIONAL TIME COSTS FOR 1 WIND TURBINE

Time interval simulation	**Discretization**	**Computational cost (s)**	
(s)	**Δ_t (s)**	**FDD**	**Symbolic Solution**
14.3	10^{-4}	21.17	4.02
0.2	10^{-4}	0.68	1.86
18.9	10^{-4}	24.97	5.5

TABLE II
COMPUTATIONAL TIME COSTS FOR 10 WIND TURBINES

Time interval simulation	**Discretization**	**Computational cost (s)**	
(s)	**Δ_t (s)**	**FDD**	**Symbolic Solution**
14.3	10^{-4}	84.78	13.25
0.2	10^{-4}	2.63	6.49
18.9	10^{-4}	100.45	20.49

TABLE III
COMPUTATIONAL TIME COSTS FOR 40 WIND TURBINES

IV. CONCLUSION

A symbolic method to solve the model of a wind turbine equipped with DFIG is described and discussed. The aim of this approach is focused on simulating a large number of wind turbines with a lower computational cost respect to the traditional method based on discretized techniques.

The symbolic method is compared with classical Finite–Difference Discretization technique for typical time–step values, varying the number of wind turbines considered in the simulation. These comparisons have been carried out under different real wind speed conditions and transient disturbances, such as voltage dips. Real wind speed data have been collected at hub height of a Spanish wind farm and filtered through an equivalent wind speed model. The results of the comparisons provide a good agreement between the proposed symbolic method and FDD technique along the whole simulation time. Furthermore, the symbolic method presents clearly advantages in terms of computational time requirements when large time simulation period and small integration-time step are considered. Consequently, the proposed symbolic method is highly suitable to simulate individually a substantial number of wind turbines facing different wind speed profiles and under transients events.

V. LEARNING OBJECTIVES

The main learning objective of this work is based on how a DFIG wind turbine can be modelled in a linearized way in order to minimize the computational cost required for its simulation. The tools that have been developed for it, can be employed for other type of simulations.

ACKNOWLEDGMENT

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