A vortex based BEM-like algorithm accounting for wake rotation

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Abstract

The importance of the pressure drop due to wake rotation for wind energy applications was pointed out by Sharpe [7]. This raised questions about the possibility to beat the Betz-Joukowski limit. Prior investigations for propeller applications are found in the work of McCutchen [6]. The inclusion of swirl in the classical actuator disk theory introduces a singularity which can be linked to the singularity of the root vortex. The effect of swirl and the regularization of the root vortex was investigated e.g. by Wood [9] using momentum theory. Wood used far wake relations derived for a high tip-speed ratio case, so that the helical pitch does not include the possible effect of the azimuthal velocity. Øye, in the work of Madsen et al. [5], used vortex theory to account for wake rotation in the case of a constant circulation disk. In this work, the helical pitch includes the effect of the azimuthal velocity. Sørensen and van Kuik [8] studied the constant circulation disk with a regularized root vortex using momentum analysis. The authors also discussed the different analyses found in the literature. Corrections to BEM algorithm to include the effect of wake rotation were suggested by Madsen et al. [4] based on comparisons with actuator disk simulations.

The current paper is based on the cylindrical actuator disk model of Joukowski [3] which has received recent interest by the authors [1]. The study showed that a superposition of cylindrical models under infinite tip-speed ratio gives results identical to a standard stream-tube BEM formulation. Under finite tip-speed ratio, the model contains a term which is not accounted for in the standard Blade Element Momentum (BEM) algorithm. This extra term was identified as the contribution from the pressure drop due to the wake rotation [2].

The current paper summarizes the implementation of the BEM-like algorithm which can be derived from the superposition of cylinders. The implementations includes corrections for finite number of blades and high-thrust coefficients. The implementation is really close to a BEM formulation and similar computational times are achieved. The model is compared to the BEM-correction derived by Madsen to account for wake rotation. Results are also compared to actuator disk simulations.

Preliminary results are shown in Figure 1.

References

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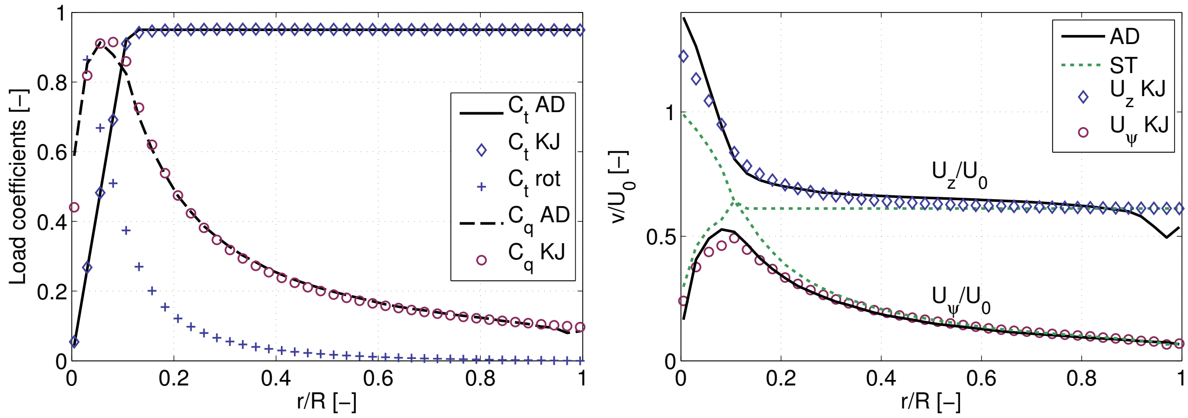

(a) (b)

Figure 1: Loads (left) and velocities (right) for a prescribed thrust distribution. Results for the current vortex model (labeled KJ) are compared to actuator disk (AD) simulations and stream-tube (ST) theory.

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